الأبحاث المنشورة(1965-1974)

فى مجال صناعة بناء وإصلاح السفن

للأستاذ الدكتور محمد عبد الفتاح شامة

Published Papers (1965-1974)

on Shipbuilding and Ship Repair

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Prof. Dr. M. A. Shama

1- "A Design Study of a Numerically Controlled Frame Bending Machine", RINA. (UK-1965), Shama, M. A. and Miller, N. S., (90%)

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FACULTY OF ENGINEERING

PLASTIC BENDING OF SHORT MILD STEEL BEAMS

By

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Lecturer, Marine Engineering Department

Reprinted from Bulletin of the Faculty of Engineering, Alexandria University — Vol. IV — 1965

ALEXANDRIA UNIVERSITY PRESS
1966

PLASTIC BENDING OF SHORT MILD STEEL BEAMS

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INTRODUCTION

Cold bending is potentially the most economic method used for forming plates and sections. It is widely used in the shipbuilding and aircraft industries.

In order to study the behaviour of mild steel beams when they are bent in the plastic range of the material, it is necessary to refer to the stress-strain diagram of the material since it is the key to all forming problems.

This paper deals with the physical nature of the 3-point bending method. The relationships between the applied load or bending moment and the different variables associated with a beam simply supported and centrally loaded were established in the elastic and plastic regions. The spring back is also investigated in terms of the initial and final deformations as well as in relation to the applied force.

It is to be noted that there is no definite theory which predicts the precise behaviour of mild steel in the plastic range of the material (14) since all the theories dealing with plastic behaviour are either based on simplifying assumptions (ideal plastic material, —— etc.) or based on empirical relationships governing the factors which affect the plastic flow (Tresca Yield-Criterion, ... etc.).

As a result, most of the analysis will be chiefly qualitative rather than quantitative. Tests were carried out on rectangular section beams and some of the results are presented here only for the sake of comparison.

The text describes the problem in general terms and the mathematical treatment is given in the appendix.

Stress — Strain Diagram :

In forming problems, it is general practice to modify the stress-strain diagram in such a way that it consists of only two main regions as follows, see fig. (1) :

a. *Elastic Region*

In this region, all the stresses and strains are assumed to vanish upon unloading. The stress-strain relationship is as follows :—

$$\sigma = E \epsilon \qquad \ldots \ldots (1)$$

b. *Strain Hardening Region*

b) *Strain Hardening Region* :

This region represents the plastic state when the time factor is not important at room temperature. The slope of the line *AB* in fig. (1) depends on the maximum strain attained and normally decreases as the strain increases. In this region, when a member is loaded so that the applied stress is σ_1 as shown in fig. (1), the correspoding strain is ϵ_1. However, when the load is removed, the unloading curve is assumed to follow the line *CD*, which is parallel to the elastic line *OA* (1). This unloading process is entirely elastic and is called the "Elastic Recovery" or "Spring back". Obviously, the magnitude of the spring back depends on the stress state and the shape of the stess-strain diagram (2).

From Fig. (1), we have :—

$$\epsilon_1 = \epsilon_p + \epsilon_s \qquad \ldots \ldots (2)$$

where : ϵ_1 = total strain before unloading

ϵ_p = permanent strain after unolading

ϵ_s = spring back

The stress-strain relationship for the tension zone can be written as follows :

$$\sigma = \sigma_y + (\epsilon - \epsilon_y) \tan \alpha \qquad \ldots \ldots (3)$$

where σ = stress in the strain hardening region i.e. $\sigma \geqslant \sigma_y$

ϵ = strain in the strain hardening region i.e. $\epsilon \geqslant \epsilon_y$

σ_y = yield stress of the material

ϵ_y = yield strain of the material

tan. α = rate of strain hardening

Stress and Strain States :—

In order to satisfy the compatibility condition, the strain distibution across the beam depth is assumed to be linear (3), even when bending is carried into the plastic range of the material, i.e. :

$$\epsilon = \frac{y}{\rho} \qquad \ldots \ldots \quad (4)$$

where ϵ = strain at a distance y from the neutral axis.

y = distance from the neutral axis.

ρ = radius of curvature attained by the neutral axis.

The stress pattern is obtained from equations (3) and (4) and is given by :

$$\sigma = \sigma_y + \left(\frac{y}{\rho} - \epsilon_y\right) \tan. \alpha \qquad \ldots \ldots \quad (5)$$

Equation (5) gives the stress distribution across the beam depth when it is simply supported and centrally loaded, see fig. (2).

In the elastic range, the stress and strain distributions are linear. When the load is increased such that the section is partially elastic and partially plastic, the stress distribution is as shown in fig.(2). Finally, full plasticity will take place when the load is increased in such a way that plastic flow spreads almost to the neutral axis of the section. This neutral axis is different from the centroidal axis. The latter is used only when elastic bending is considered while the former is used for inelastic bending (4), However, both axes are obtained from the following conditions :

$$\Sigma F = o \qquad \text{and} \qquad \Sigma M = o$$

where F = normal force and M = Bending moment

Bending Moment — Curvature Relationship :—

This relationship is obtained from the bending moment — extreme fibre strain relationship on the assumption that plane sections before bending remain plane and normal to the neutral axis — i.e :

$$\epsilon = \frac{y}{\rho}$$

Timoshenko (5) gives a method whereby the bending moment - extreme fibre strain relationship could be determined when the true stress-strain diagram is used. The analysis is complicated, however, and Nadai (6) gives an approximate method based on a modified stressstrain diagram as shown in fig. (1). The accuracy of this method depends entirely on the accuracy of approximation of the stress-strain diagram. In the following analysis, Nadai's method has been used.

The bending moment M is given by :

$$M = \int_{0}^{A_e} \sigma . dA . y + \int_{0}^{A_p} \sigma . dA . y \quad .. \quad .. \quad (6)$$

where σ = stress at a distance y.

y = distance from the neutral axis.

A_e = area of elastic core.

A_p = area of plastic zone.

See Appendix (1).

If it is assumed that ϵ_1 is the extreme fibre strain, then the bending moment-extreme fibre strain relationship becomes :

$$M = M_e + Z_p (\sigma_y - \epsilon_y . \tan \alpha) + (\frac{I_p}{y}) \epsilon . \tan . \alpha \quad .. \quad (7)$$

where M_e = Bending moment of the elastic core.

Z_p = plastic modulus of the section.

Having established this relationship, it is then possible to determine the $M - \frac{1}{\rho}$ relationship by substituting $\frac{y}{\rho}$ for ϵ as follows :

$$M = M_e + Z_p (\sigma_y - \epsilon_y \tan. \alpha) + (\frac{Ip}{\rho}) \tan \alpha \quad .. \quad (8)$$

This equation gives the required bending moment for any value of the curvature when the yield stress σ_y, yield strain ϵ_y and the rate of strain hardening tan α are known, see fig. (4).

Load-Central Deflection Relationship :—

If the three-point bending method, used for forming sections, could be approximated by a beam simply supported and centrally loaded, the central deflection could be calculated only when the beam is loaded into the elastic range of the material (i e) when:

$$M \leqslant M_y$$

where M_y = yield moment.

In this region, the total deflection is due to a combination of bending and shear, i.e.

$$\Delta t = \Delta_b + \Delta_s$$

where Δ_t = total deflection

Δ_b = deflection due to bending

Δ_s = deflection due to shear

$$\Delta t = \frac{W. L^3}{48EI} + \frac{K. W. L}{4AG}$$

where : W = applied load.

I = moment of inertia of the beam section.

L = beam span.

A = beam cross-sectional area.

G = modulus of rigidity.

E = Young's modulus.

K = shape factor (a constant depending on the shape of the cross-section).

Hence the relationship between the applied load "W" and the total deflection Δ_t in the elastic range, is given by :

$$\frac{W}{\Delta_t} = \frac{1}{\frac{L^3}{48EI} + \frac{KL}{4AG}} \quad \ldots \ldots (9)$$

Practically $E = 2.6\, G$, when Poission's ratio $\mu = 0.3$. Therefore, equation (9) becomes :

$$\frac{W}{\Delta_t} = \frac{4E}{\frac{L^3}{12I} + \frac{2.6\,KL}{A}} \quad \ldots \ldots (10)$$

For a rectangular section of depth and thickness d and b respectively we have :

$$I = \frac{bd^3}{12}, \quad A = b.d, \quad K = 1.2$$

$$\therefore \quad \frac{W}{\Delta_t} = \frac{4Eb}{\left(\frac{L}{d}\right)^3 + 3.12\left(\frac{L}{d}\right)} \quad \ldots \ldots (11)$$

From equation (11), it is shown that as the $\frac{\text{span}}{\text{depth}}$ ratio decreases, the deflection due to shear becomes more significant and since relatively deep beams are considered, the deflection due to shear should be taken into account, see fig. (5).

Timoshenko (7) gives another formula for elastic deflection of rectangular section beams, simply supprted and centrally loaded, as follows :

$$\frac{W}{\Delta_t} = \frac{4\,Eb}{\left(\frac{L}{d}\right)^3 + 2.85\left(\frac{L}{d}\right) - 0.85} \qquad \ldots \ldots (12)$$

The difference between (11) and (12) is not significant when the ratio of $\frac{L}{d}$ is in the range of from 3—10 (the working range of cold forming of deep sections), but becomes very significant when $\frac{L}{d}$ ratio is of the order of unity (i.e.) when very short beams are considered.

Further, Timoshenko (8) gives a semi-graphical method whereby the central deflection can be determined when the beam is loaded into the elasto-plastic region. B.G. Neal (4) also gives another method whereby the central deflection can be determined using the simple theory of plastic bending. The maximum deflection calculated by this method is obtained when the fully plastic moment is attained at midspan.

The total deflection depends not only on the load but on the loading rate and on the strain history of the material as well as the physical properties. Thus there is no method available at present, whereby the central deflection can be predicted when the beam is bent well into the strain hardening range of the material. J.W. Roderick (9) gave the following statement : "In attempting an accurate determination of deflection of a partially plastic joist, it is not possible to derive mathematical expressions of manageable proportions directly since the cross section is not a simple geometrical shape, and because of the need to take the true stress-strain relationship into account".

J.W. Roderick and I.H. Phillips (10) gave a historical review of investigations bearing upon the several forms of the simple theory of bending. It was pointed out that for rolled steel sections much further information about the likely variation in properties across the section will be necessary before it is possible to apply any of these theories.

In the same way as before, the relationship between the applied bending moment and the slope at the supports could be obtained. The total slope at the support in the elastic range of the material for a rectangular section is given by :

$$\frac{M}{\theta_t} = \frac{EI}{\frac{L}{4} + \frac{6.24\,I}{AL}} \qquad \ldots \ldots (13)$$

Equation (13) takes into account the slope due to shear.

Spring Back

Spring back may be defined as the process of elastic recovery which occurs when the applied load is removed.

In the case of three-point bending, spring back can be defined in two ways as follows :—

a) Spring back associated with central deflection or slope at the support.

b) Spring back associated with strains or curvatures.

A previous knowledge of the amount of spring back would be very helpful in predicting the required deformation necessary for forming.

The spring back, unfortunately, depends on the shape of the true stress-strain diagram, which varies over a wide range. Consequently an empirical relationship, whereby the spring back can be predicted, seems to be the only solution and can only be achieved by conducting a series of tests.It is expected that the spring back data will not form a definite relationship although the theoretical relationship between the applied load and the spring back is one valued and independent of time (i.e.) a linear relationship (3).

The spring back problem for sheet metal has been studied before on the assumption that the true stress-strain diagram of the material is known. Schroeder (11) gives a method whereby the spring back in its angular phase can be determined graphically and analytically. F.J. Gardiner (3) gives extensive test data for some elastic materials including mild steel as well as a simplified mathematical solution. He shows that spring back data form a band bounded by a lower and an upper limit.

A simple approach for predicting the spring back in the case of a beam simply supported and centrally loaded is presented as follows :

a) Spring back associated with central deflection

In the elastic range, when the beam is loaded and then unloaded, there will be no permanent deflection (i.e.) the total deflection will be entirely recovered after unloading; see fig. (6) :

$$\therefore \quad \Delta_t = \delta_s \qquad \ldots \ldots (14)$$

where : δ_s = spring back

Δ_t = imposed central deflection.

On the other hand, when the beam is bent into the strain hardening region, the total deflection Δ_t consists of two components as follows :

$$\Delta_t = \delta + \delta_s \qquad \ldots \ldots (15)$$

where : δ = permanent deflection after spring back

It can be shown that the relationship between the applied load W and the associated spring back in the central deflection i.e. W—δ_s relationship is similar to W—Δ_t relationship as obtained from elastic bending.

$$\therefore \frac{W}{\delta_s} = \frac{4E}{\frac{L^3}{12I} + \frac{2.6KL}{A}} \qquad \ldots \ldots (16)$$

which is a linear relationship. (i.e) $\frac{W}{\delta_s} = C$

where : C is a constant depending on the geometrical properties of the section. For a rectangular section beam, W—δ_s rlationship is given by :

$$\frac{W}{\delta_s} = \frac{4Eb}{(\frac{L}{d})^3 + 3.12\,(\frac{L}{d})} \qquad \ldots \ldots (17)$$

This is described as the recovered part of the curvature which takes place after unloading.

A method of evaluating this phase of spring back has been presented by R.G. Sturm and B.J.. Fletcher (12). The relationship between the bending moment and associated spring back in curvature is given by :

$$\frac{1}{\rho_1} - \frac{1}{\rho_2} = \frac{M}{EI} \qquad \ldots \ldots \quad (18)$$

where : $\frac{1}{\rho_1}$ = curvature of neutral axis before spring back.

$\frac{1}{\rho_2}$ = curvature of neutral axis after spring back.

M = applied bending moment.

However, this relationship could be obtained directly from the bending moment curvature relationship, see fig. (7).

From Fig. (7) we have :

$$\frac{1}{\rho_s} = \frac{M}{EI} \qquad \ldots \ldots \quad (19)$$

where : $\frac{1}{\rho_s}$ = spring back in curvature

$$= \frac{1}{\rho_1} - \frac{1}{\rho_2} \qquad \ldots \ldots \quad (20)$$

From the preceeding theoretical analysis, it is clear that in order to calculate the applied load or bending moment required to produce a certain amount of deformation (central deflection, curvature, ... etc.) it is necessary to know the precise stress-strain diagram of the material, (i.e.) yield stress, yield strain and rate of strain hardening (13) as well as the geometrical properties of the section.

However, it is expected that the calculated deformations or forces will contain some errors, since the methods used are based on simplifying assumptions (beam simply supported, linear strain hardening, ... etc).

In order to assess the degree of accuracy of these calculations, laboratory tests were carried out. The results of these tests are shown in figs. (8), (9), (10).

The variation in the yeild stress for shipbuilding mild steel was found to be between 13.0 — 18.0 tons/inch^2 with a mean deviation of 16.15% based on a mean yield stress of 15.5 tons/inch^2.

The maximum deviation in the flexural rigidity EI was found to be 11.8%

The maximum deviation in the slope of W— Δ curve was found to be 33.33%.

The maximum deviation in the slope of M —θ_S curve was found to be 36.8%.

The maximum deviation in the slope of W — δ curve was found to be 15%.

The maximum deviation in the slope of Δ — δ curve was found to be 3.85%.

These errors indicate clearly the significant deviation between the theoretical and experimental results, even when bending is carried in the elastic range of the material. Consequently these tests have obviated the possibility of using these methods to predict the behaviour of mild steel beams when they are bent in the plastic range of the material.

The detailed analysis of these tests and their results will be published in a separate paper.

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APPENDIX (I)

In order to establish a theoretical relationship between the applied load or bending moment and the different variables associated with this method of bending, the following assumptions are used;

a) Stress-strain diagram of the material is as shown in fig. (1)

b) Stress-strain diagram is identical for both tension and compression.

c) Plane sections before bending remain plane after bending and normal to the neutral axis,

$$\text{i.e. } \epsilon = y/\rho \qquad \ldots \quad \ldots \quad (1)$$

where y = distance from the neutral axis.

$1/\rho$ = curvature attained by neutral axis.

ϵ = strain attained at the distance y.

d) Only symmetrical sections are dealt with.

e) The plane of bending coincides with the axis of symmetry.

f) The neutral axis passes through the centroid of the section.

g) The material is homogenious and isotropic.

It is necessary to consider both the elastic and plastic regions since although the permanent deformation takes place in the latter region, the unloading process (spring back) is purely elastic.

A) *Bending moment-Extreme fibre strain relationship.*

$$\text{The bending moment } M = \int_{o}^{Ae} \sigma.dA.y + \int_{o}^{A\rho} \sigma.dA.y \qquad (2)$$

where : A_e = area elastic core.

A_p = area of plastic zone.

σ = stress at a distance y.

From the strees-strain diagram Fig. (1) we have :

$$\sigma = E.\epsilon \text{ .... for } 0 < \epsilon \leqslant \epsilon_y \quad \text{.. ..} \quad (3)$$

$$\sigma = \sigma_y + (\epsilon - \epsilon_y) \tan a \text{ for } \epsilon_y \leqslant \epsilon \leqslant \epsilon_1$$

$$\therefore \; M = \int_o^{A_e} E.\epsilon.\, dA.y + \int_o^{A_p} [\sigma_y + (\epsilon - \epsilon_y) \tan a]\, dA.y \quad \text{.. ..} \quad (4)$$

Substituting equation (1) into (4), we get :

$$M = \frac{E}{\rho} I_e + (\sigma_y - \epsilon_y \tan a). \int_o^{A_p} A_p \bar{y}_p + \frac{I_p}{\rho} \tan a \quad (5)$$

where : I_e = moment of inertia of the elastic core.

I_p = moment of inertia of the plastic zonees.

$\bar{y}_p$ = distance of centroids of plastic zones from the neutral axis.

$$M = M_e + (\sigma_y - \epsilon_y \tan a). [A_1\bar{y}_1 + A_2\bar{y}_2] + \frac{I_p}{h} \epsilon. \tan a \quad \text{..} \quad (6)$$

where : M_e = bending moment of the elastic core.

h = distance of extreme fibres from neutral axis.

A_1 and A_2 = area of tensile and compressive plastic zones respectively.

$\bar{y}_1$ and $\bar{y}_2$ = centroids of A_1 and A_2 respectively.

Equation (6) gives the required bending moment for any section bent into the elasto-platic region. Using equations (6) & (1), the bending moment-curvature relationship could be obtained as follows :

$$M = M_e + (\sigma_y - \epsilon_y \tan a). [A_1\bar{y}_1 + A_2\bar{y}_2] + \frac{I_p}{\rho} \tan a \quad \text{.. ..} \quad (6a)$$

where : Zp = plastic modulus of the section
Equation (6a) could be written as :

$$M = Me + (\sigma_y - \epsilon_y \tan \alpha).\ Zp + \frac{Ip}{\rho} \tan \alpha \quad \ldots \quad (7)$$

Equations (6) and (7) are similar and could be represented by 3-different regions, namely :

1. *Elastic region.*

This is represented by the elastic line OA, as shown in fig. (4). The bending moment at the end of this region is obtained when :

$$\epsilon_1 = \epsilon_y \quad \text{or} \quad \frac{1}{\rho} = \frac{1}{\rho_y}$$

The characteristics of this region are given by the following equations :

a) Slope of the elastic line

$$i)\ \frac{M}{\epsilon} = \frac{EI}{h} = EZ_e \quad \ldots \ldots \quad (8)$$

$$ii)\ M / \frac{1}{\rho} = EI \quad \ldots \ldots \quad (9)$$

b) Load at the end of this region

$$M_y = \sigma_y Z_e \quad \ldots \ldots \quad (10)$$

When the beam is loaded and then unloaded, there will be no permanent strains and the beam will take its original shape.

2. *Elasto-plastic region*

This region is represente dby the curve AB, as shown in fig. (4). It represents the transition stage between the elastic region, ending at point A, and the strain hardening region, starting at point B, In this region, there will exist a small amount of permanent strains upon unloading.

3. *Strain hardening region*

This region is represented by the straight line BD, as shown in fig. (4) and is attained when :

$$\epsilon >> \epsilon_y \quad \text{or} \quad \frac{1}{\rho} >> \frac{1}{\rho_y}$$

(i.e.) when $M_e \doteqdot o$ in equation (7)

where : $\dfrac{1}{\rho_y}$ = curvature attained when M_y is applied.

$$M = (\sigma_y - \epsilon_y \tan a).\ Zp + \frac{I_p}{h}\ .\ \epsilon \tan a \quad \ldots \quad \ldots \quad (11)$$

In the case of $(M - \dfrac{1}{\rho})$ relationship, equation. (11) becomes:

$$M = (\sigma_y - \epsilon_y \tan a)\ Zp + \frac{I_p}{\rho} \tan a \quad \ldots \quad \ldots \quad (12)$$

Equations (11) and (12) are linear equations and have the following characteristics :

a) *Slope*

i) $$\frac{dM}{d\epsilon} = \frac{I_p}{h} \tan. a = Ze \tan a \quad \ldots \quad \ldots \quad (13)$$

ii) $$\frac{dM}{d\left(\frac{1}{\rho}\right)} = I_p \tan a = I \tan a \quad \ldots \quad \ldots \quad (14)$$

b) Intercept on the M—axis = M_o = constant

$$M_o = Z_p(\sigma_y - \epsilon_y \tan\alpha) \quad \dots \quad \dots \quad (15)$$

Equation (12) could be written as follows :

$$M = \sigma_y Z_p + \frac{Ip}{\rho}\tan\alpha - Zp.\epsilon_y.\tan\alpha$$

$$= M_p + I_p \tan\alpha\left(\frac{1}{\rho} - \frac{Z_p}{I_p}\epsilon_y\right)$$

But $Zp = KZ_e$

where : K = Shape factor $= \frac{Z_p}{Z_e}$

and I_p = I when full plasticity is attained.

$$\therefore M = M_p + I\tan\alpha\left(\frac{1}{\rho} - \frac{1}{\rho_p}\right) \quad \dots \quad \dots \quad (16)$$

where : $\frac{1}{\rho_p}$ = curvature attained when Mp is applied.

The bending moment at the point "B" implies that the fully plastic moment has been attained and plasticity has spread through the beam depth until it has reached the neutral axis of the section.

In this region, when the beam is loaded and then unloaded, there will be a permanent set of considerable magnitude compared to the elastic spring back .

When the material behaves in an ideal plastic fashion (i.e.) when $\tan\alpha = o$.

$$M = Mp = \sigma_y Z_p \quad \dots \quad \dots \quad (17)$$

which is the plastic moment

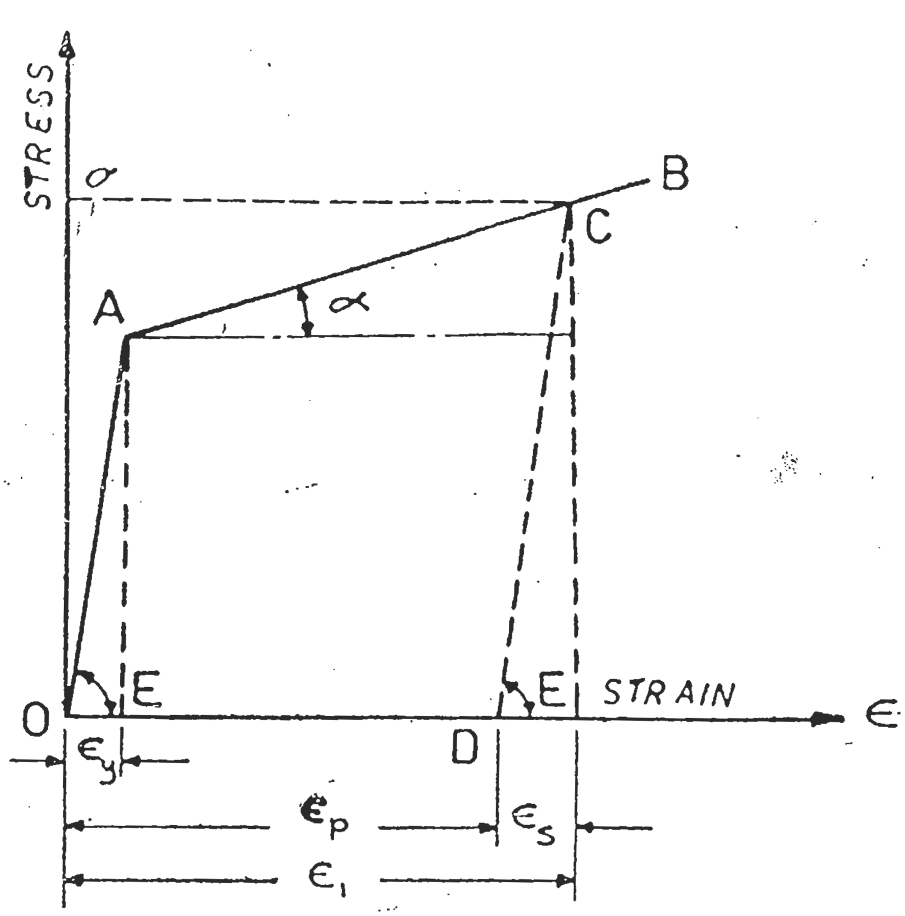


Fig. (1) Stress —Strain Diagram

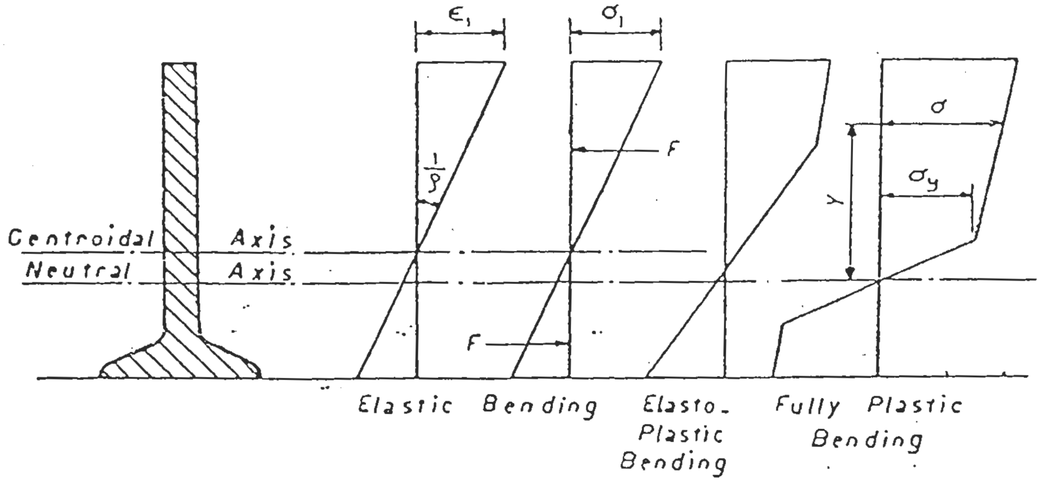


Fig. (2) Znelastic Stress Distribution

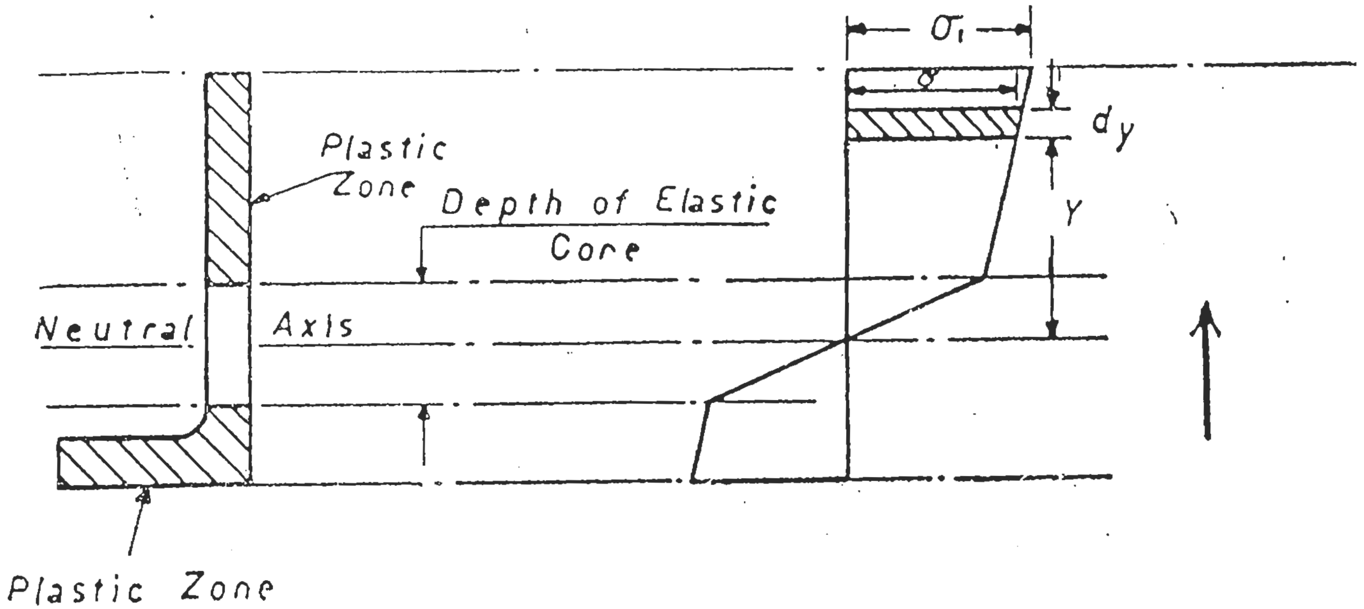


Fig. (3) Elastic and Plastic Zones

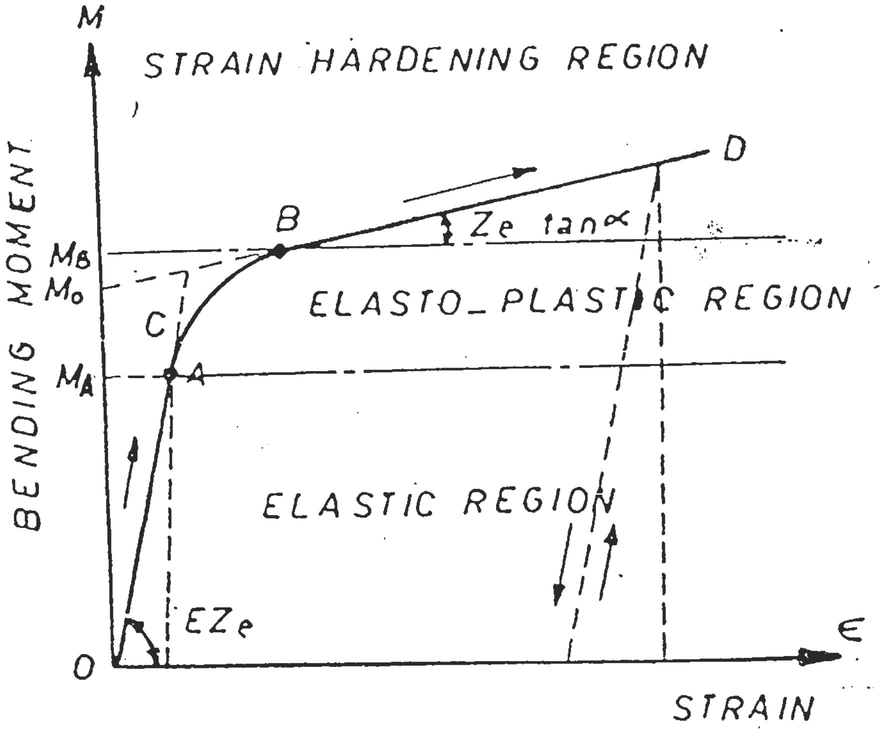


Fig. (4) M – ∈ Curve

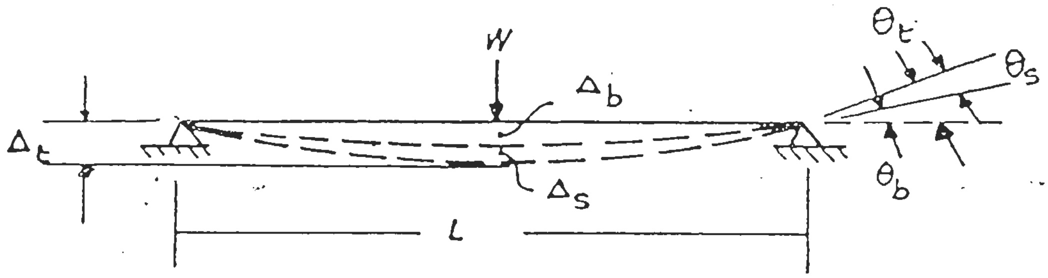


Fig. (5) Bending and shear Deflections

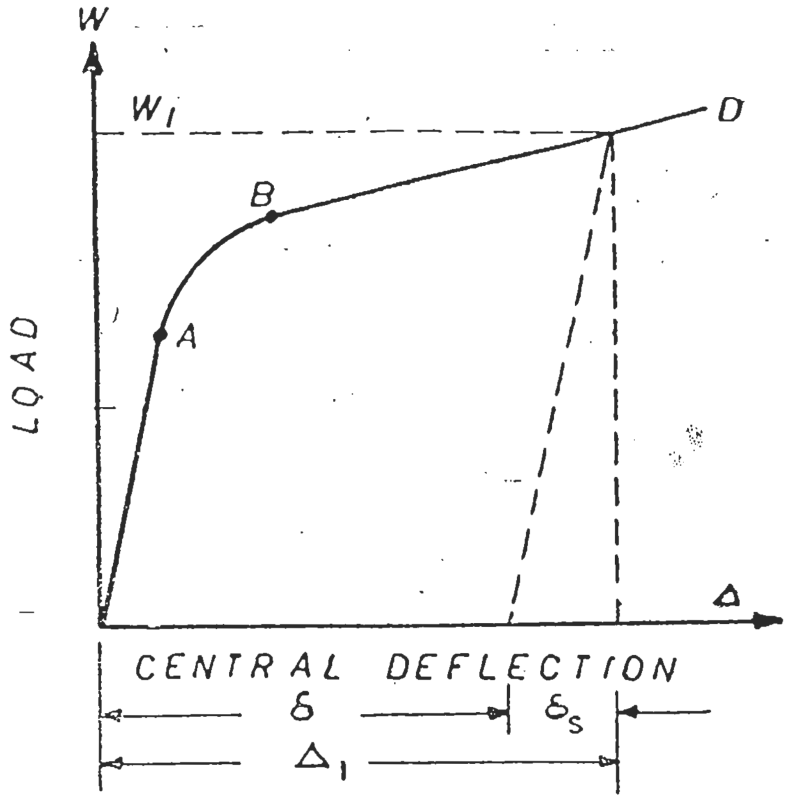
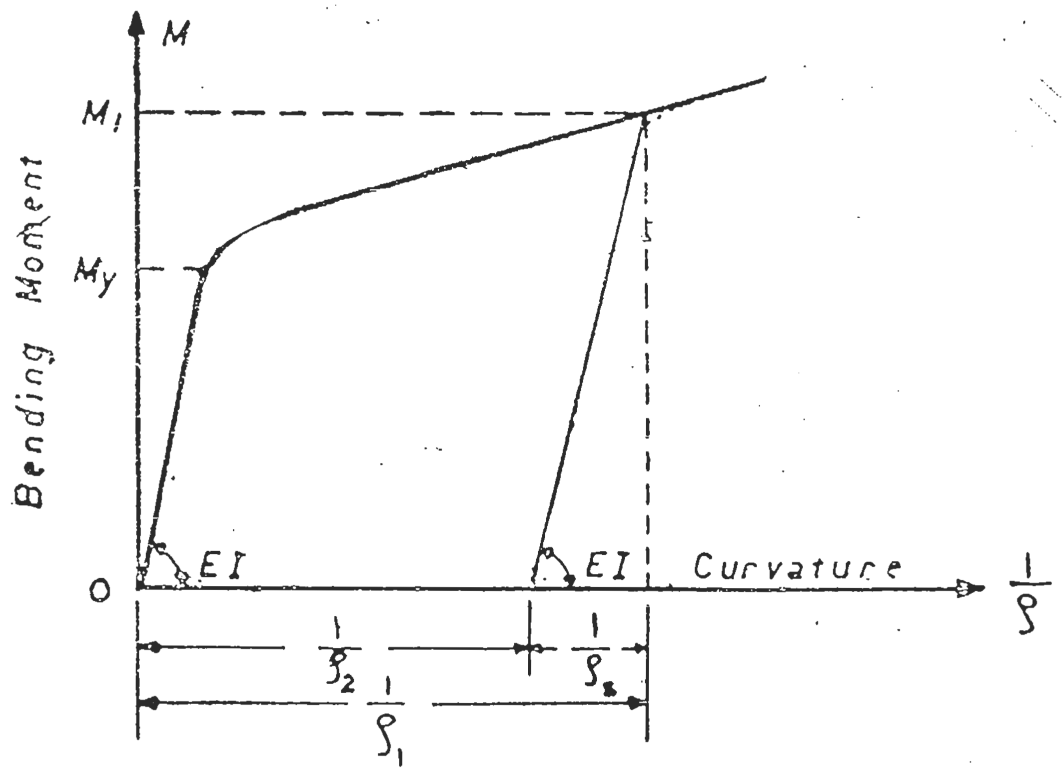


Fig. W — Δ Curve



Feg. (7) M — $\frac{1}{\rho}$ Curve

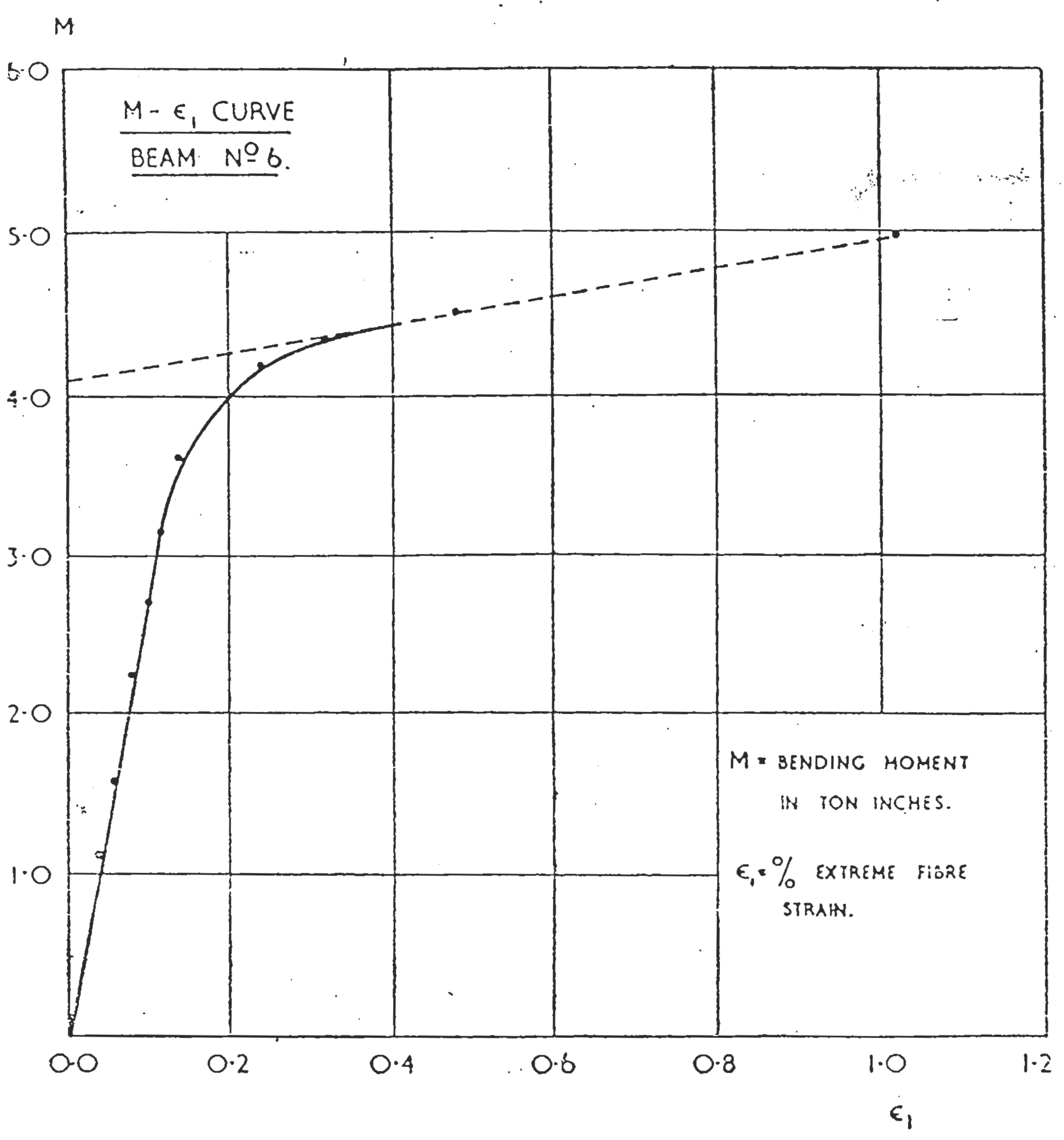


Fig. (8) $M-\epsilon_1$ Curve

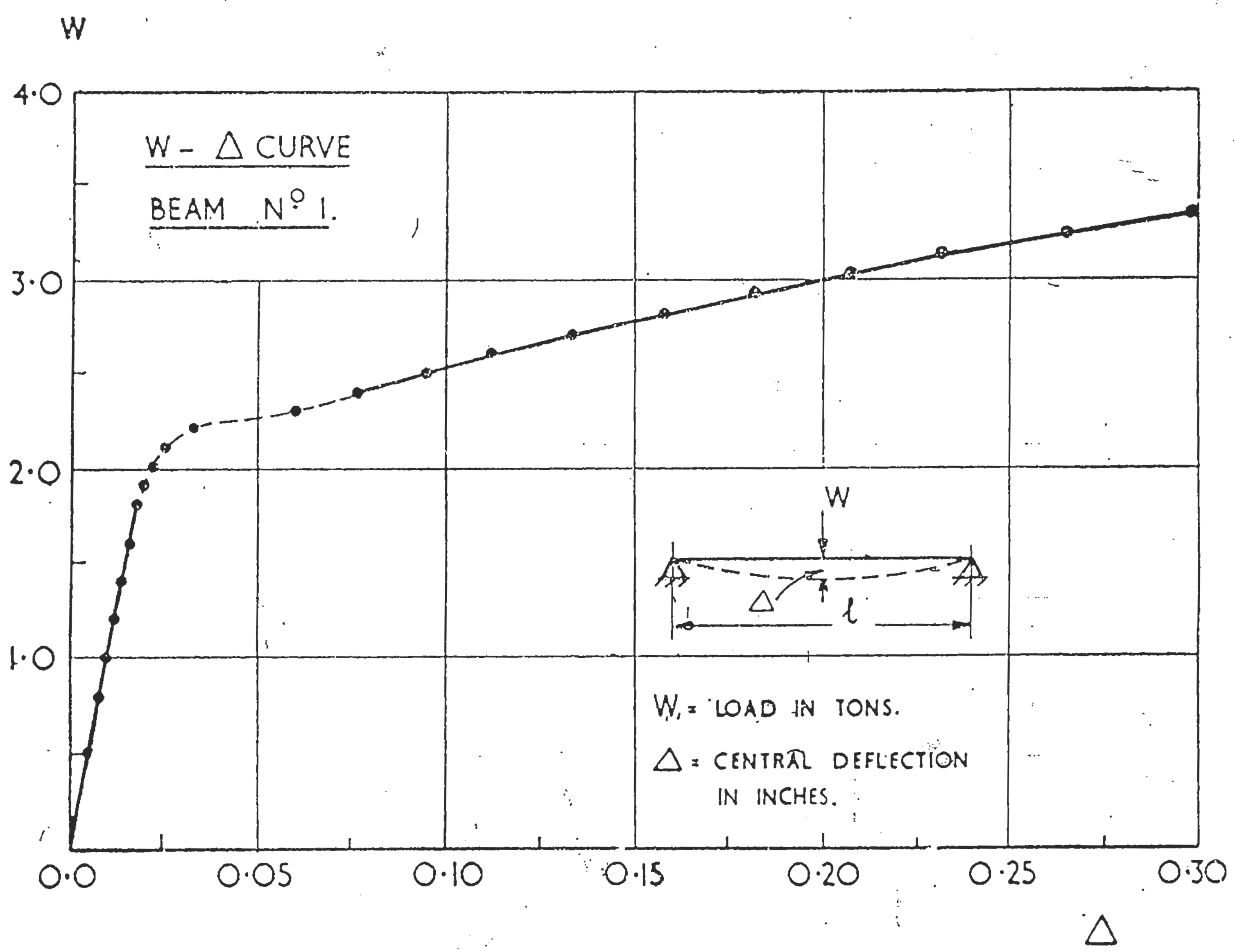
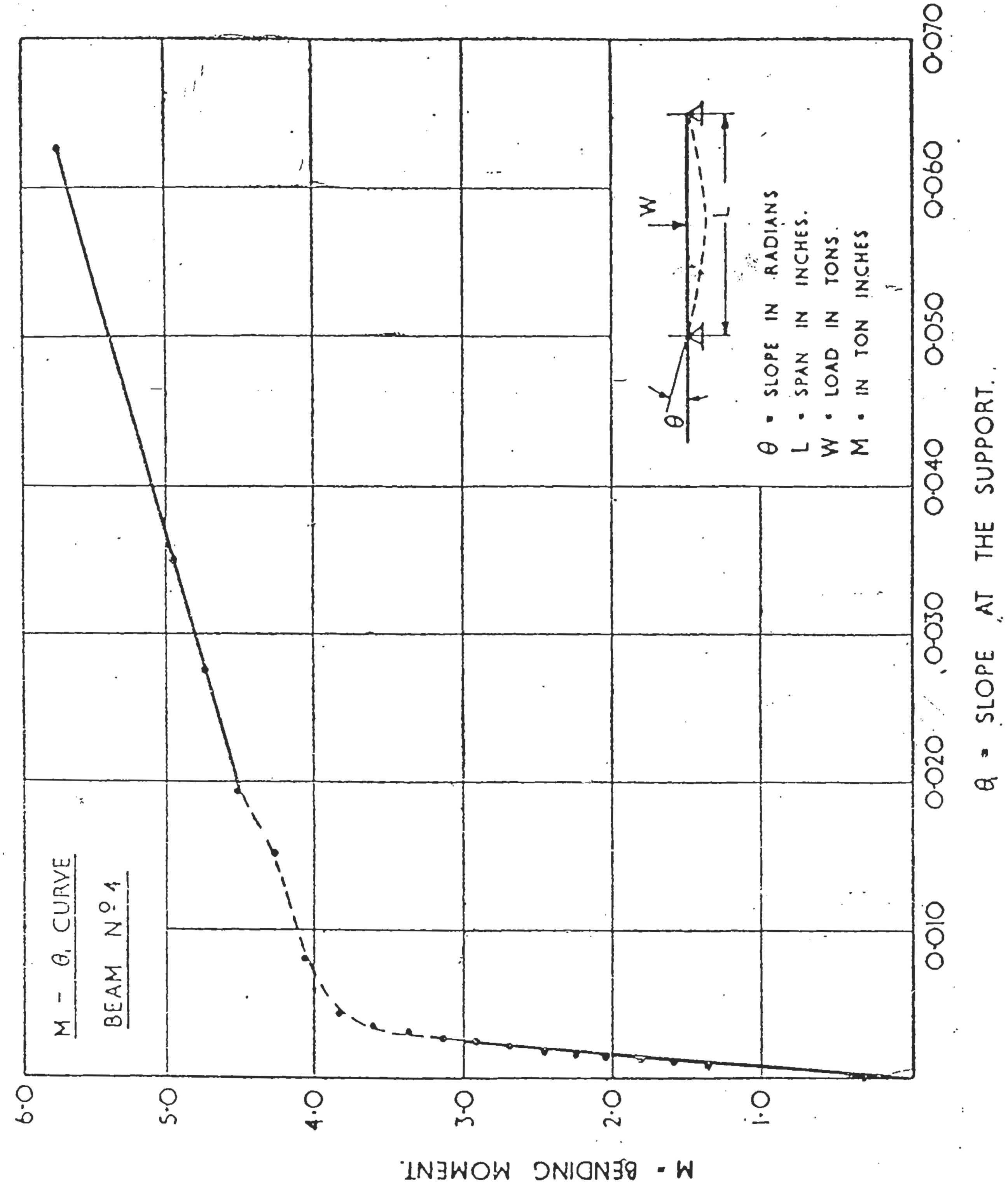


Fig. (9) W – Δ Curve



M - θ1 CURVE
BEAM No 4
M - BENDING MOMENT
θ1 - SLOPE AT THE SUPPORT.
θ - SLOPE IN RADIANS
L - SPAN IN INCHES.
W - LOAD IN TONS.
M - IN TON INCHES
W
L
θ
1·0
2·0
3·0
4·0
5·0
6·0
0·010
0·020
0·030
0·040
0·050
0·060
0·070